

Use of Complex Impedance

The handling of the [impedance](#) of an AC circuit with multiple components quickly becomes unmanageable if sines and cosines are used to represent the voltages and currents. A mathematical construct which eases the difficulty is the use of complex exponential functions. The basic parts of the strategy are as follows:

Math relationship underlying the technique	$e^{j\omega t} = \cos \omega t + j \sin \omega t$	Euler relation	Polar form of complex number
The real part of a complex exponential function can be used to represent an AC voltage or current.	$V = V_m \cos \omega t$ $I = I_m \cos(\omega t - \phi)$	represent by \Rightarrow $V = V_m e^{j\omega t}$ $I = I_m e^{j[\omega t - \phi]}$	
The impedance can then be expressed as a complex exponential.	$Z = \frac{V_m}{I_m} e^{-j\phi} = R + jX$	Impedance combinations	Phasor diagrams
The impedance of the individual circuit elements can then be expressed as pure real or imaginary numbers.	R $\frac{-j}{\omega C}$ $j\omega L$	RL and RC combinations	Example for parallel elements

[Index](#)
[AC circuit concepts](#)
[HyperPhysics***** Electricity and Magnetism](#)

R Nave

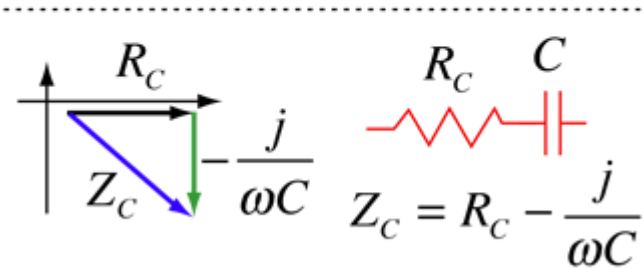
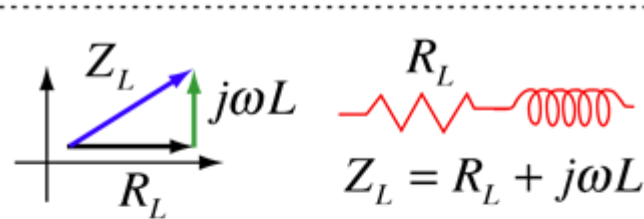
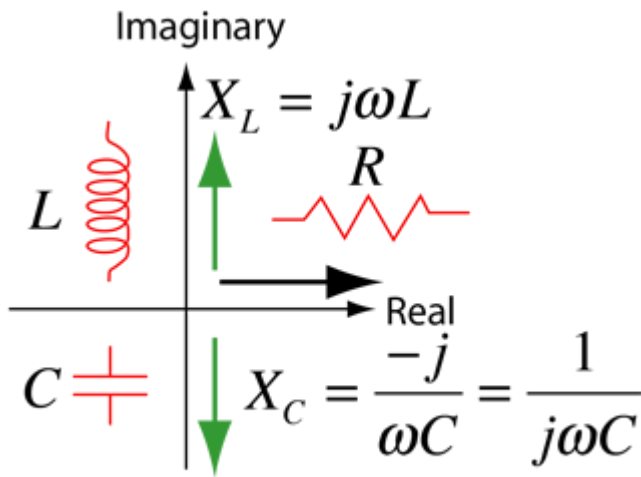
[Go Back](#)

Complex Impedance for RL and RC

Using [complex impedance](#) is an important technique for handling multi-component AC circuits. If a complex plane is used with resistance along the real axis then the reactances of the [capacitor](#) and [inductor](#) are treated as imaginary numbers. For series combinations of components such as RL and RC combinations, the component values are added as if they were components of a [vector](#). Shown here is the cartesian form of the complex impedance. They can also be written in [polar form](#). Impedances in this form can be used as building

[Index](#)
[AC circuit concepts](#)

blocks for calculating the impedances of combination circuits like the RLC parallel circuit.



[Impedance](#) [AC behavior of inductor](#) [AC behavior of capacitor](#)

[HyperPhysics](#)***** [Electricity and Magnetism](#)

R Nave

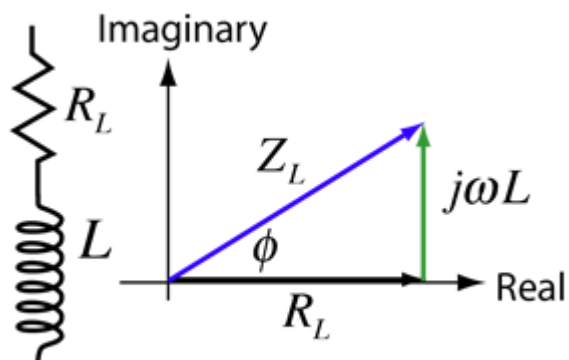
[Go Back](#)

Complex Impedance for RL and RC

This depicts the [phasor diagrams](#) and [complex impedance](#) expressions for RL and RC circuits in polar form. They can also be expressed in [cartesian form](#).

[Index](#)

[AC
circuit
concepts](#)



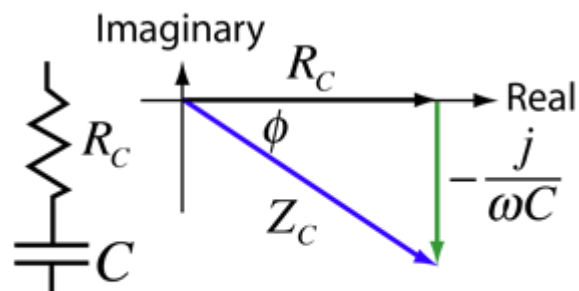
Cartesian form: $Z_L = R_L + j\omega L$

Polar form: $Z_L = |Z_L| e^{j\phi}$

where

$$|Z_L| = \sqrt{R_L^2 + \omega^2 L^2}$$

$$\phi = \tan^{-1} \frac{\omega L}{R_L}$$



Cartesian form: $Z_C = R_C - \frac{j}{\omega C}$

Polar form: $Z_C = |Z_C| e^{j\phi}$

where

$$|Z_C| = \sqrt{R_C^2 + \left[\frac{-1}{\omega C} \right]^2}$$

$$\phi = \tan^{-1} \frac{-1}{\omega C R_C}$$

[Impedance](#) [Polar form of complex number](#)